# COMMENTS ON THE COMMON CORE STANDARDS FOR MATHEMATICS

## March 2010 Draft K-12

**Submitted by the U. S. Coalition for World Class Math**
http://usworldclassmath.webs.com/

## EXECUTIVE SUMMARY

- The Common Core Standards for Mathematics fall short of the following goals:
    - The math standards in grades K-8 should lead to mastery of key procedures, skills and problem solving abilities that will enable students to undertake a full course in authentic algebra; and
    - The complete set of standards will fulfill the minimum mathematics requirements for 4-year universities in the United States and prepare students to take subsequent courses in mathematics without the need for remediation.
- If there are to be national standards, we would expect that such standards be world class—anything less than that is unacceptable. By virtue of the pedagogical ideas that are inherent in them, the standards may result in the adoption of severely deficient textbooks and programs that value process over content and that emphasize a student-centered and inquiry based approach.
- While some states may gain federal dollars in the short run by adopting these standards, the standards as written will potentially undermine our educational system for the next decade or more.
- We are extremely concerned over the fact that many of the standards are built around the word "Understand". A standard should not call for a student to learn to do one thing and lend itself to assess them at a different level. Standards should describe what the understanding will enable students to do procedurally and the types of problems they should be able to solve. As written, the standards in the current draft fall far short of that.
- The K-8 Common Core standards are simultaneously over- and under- ambitious. Numerous computation skills are delayed or under emphasized, while, at the same time, children are expected to "understand" sophisticated ideas and principles that many teachers do not themselves understand.
- The standards mention mental math for adding and subtracting whole numbers within 20 and fluency is required for add/sub of whole numbers within 10. However, students need to have immediate recall of their facts within 20. Starting in grade 1 will enable students to accomplish this. The word "memorize" need not be something to avoid.

- Pacing and rigor are in need of improvement—for example, exponents/powers/roots are not explicitly covered until high school. STEM students should be covering much of the high school material in middle school to avoid boredom and enable acceleration through upper level math.

- The high school standards are inconsistent with current college entrance requirements. Students won't have the foundation for the study of College Algebra with these standards in place, nor will many students meet the requirements for most four-year universities in this country, which call for three years of mathematics inclusive of Algebra 1 and 2, and geometry. Many topics are missing that should be included in an Algebra 2 course.

- The geometry standards do not appear to require students to conduct deductive proofs beyond key theorems. As such, geometry courses will be watered down, focusing on problem solving by applying formulae and general principles, but without developing the skills to set up a series of statements (with reasons) that systematically lead from given conditions to a specific conclusion.

## I. Introduction

We are responding to the request for public comments on the draft Common Core Standards for Mathematics, and appreciate the opportunity to do so. In reviewing the Common Core standards, we have done so with respect to what we believe are two overarching goals of these standards: 1) The math standards in grades K-8 should lead to mastery of key procedures, skills and problem solving abilities that will enable students to succeed in a full course in authentic algebra; and 2) The complete set of standards will fulfill the minimum mathematics requirements for 4-year universities in the United States and prepare students to take subsequent courses in mathematics without the need for remediation. With respect to the latter goal, we expect this would apply to students who are pursuing careers in STEM, as well as non-STEM students.

We believe that the Common Core Standards for Mathematics fall short of these goals. While it is true that the standards as currently written are better than some states' standards for mathematics, we do not feel that this is a legitimate criterion by which to base a review. If there are to be national standards, we would expect that such standards be world class—anything less than that is unacceptable. States are not required to adopt these standards, and a state wanting to improve its standards would do well to adopt those that are implemented in California, Massachusetts and Indiana. These standards do not offer a superior alternative to these states. In fact, by virtue of the pedagogical ideas that are inherent in them the standards may result in the adoption of severely deficient textbooks and programs that value process over content and that emphasize a student-centered and inquiry based approach.

We are extremely concerned that in the current political climate, weak or politically expedient standards will result in states taking the path of least resistance and adopt them. While states may gain federal dollars in the short run, the standards as written will potentially undermine our educational system for the next decade or more. Given this possibility, the current situation may be better than states adopting a set of standards that cannot possibly live up to its promise of excellence.

## II. Comments About the Introduction to the Draft Common Core Standards

The "Introduction to the Draft Common Core Standards" document says, "While we have used all available research to shape these documents, we recognize that there is more to be learned about the most essential knowledge for student success." The National Mathematics Advisory Panel's (NMAP) recommendations regarding algebra based on their review of research do not appear to be incorporated into these standards.

The Introduction document has sections with headings "Application of Common Core State Standards for English Language Learners" and "Application of Common Core State Standards for Students with Disabilities." Why are these sections here? If standards are to describe mathematical content that is essential, how does this fit in a standards document? These may be appropriate topics for another undertaking, but not for the development of the standards or the introduction to the standards. Are these the only groups with special needs in mathematics? If serving these groups is going to be addressed in a standards document, all protected groups should be addressed. All students should be expected to meet the standards of the essential math content. These sections, while not so stated, just by their inclusion would suggest otherwise. Extending or adjusting standards based on needs of individual students should be left to the professional judgment of the educators providing immediate services to the students and need not be addressed in a standards document such as this.

The "Common Core State Standards Initiative Standards-Setting Criteria" document states under Teachable and Learnable:

> *"The standards will not prescribe how they are taught and learned but will allow teachers flexibility to teach and students to learn in various instructionally relevant contexts."*

Neither the Introduction nor the standards themselves adhere to this stated principle. The Introduction even goes so far as to suggest that supports for learning may be based on the principles of Universal Design for Learning. These sections in the Introduction are beyond the scope of a standards document; they include too much pedagogy. This is a standards document, not a teaching guide. Pedagogy therefore has no place in this document.

## III. Comments About the Grade Content Outline

Each grade level has a narrative section prior to the presentation of the standards for that grade or topic. These sections will be referred to as the Content Outline. The Content Outline appears to be consistently and well written at least through grade six. It is refreshing to see that these sections are pedagogy free. Concerns with statements in the grade 2 and grade 4 content outlines include.

Grade 2 p. 16 states..

> *"They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers; understand and explain why the procedures work based on their understanding of base-ten notation and properties of operations; and use them to solve problems. "*

This needs to say that students will develop fluency in applying the standard algorithm to be able to compute sums and differences of three-digit numbers. The inclusion of "efficient procedures" will result in additional procedures that may confuse and obscure students' ability to solve problems. Also, as written, this does not require students to apply the standard algorithms to solve addition and subtraction problems. The emphasis is on understanding how the algorithms work. (We discuss this further in the next section.) One of the second grade standards states the following:

> *"13. Compute sums of two three-digit numbers, and compute sums of three or four two-digit numbers, using the standard algorithm; compute differences of two three-digit numbers using the standard algorithm."*

This is very clear and straight forward in requiring the use of standard algorithms. The grade content outline narrative should be as clear and straight forward.

Anything short of clearly requiring the fluent use of standard algorithms is unacceptable. What is the evidence base or research base for alternative procedures, even if considered efficient? Is this how the top scoring nations address algorithms? Is this internationally benchmarked? If so, is it benchmarked with the top or the bottom scoring countries?

Grade 4 p. 22 says..

> *"They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends."*

Same comments as given above for grade 2 apply.

One of the grade 4 standards is the following:

> *"8. Compute products of two-digit numbers using the standard algorithm, and check the result using estimation. "*

This is very clear and straightforward in requiring the use of standard algorithms. The grade content outline narrative should be as clear and straight forward.

While the standards themselves require the use of standard algorithms, as they well should, this requirement is not reflected in the grade content outlines through grade 6. Why are the standard algorithms being kept secret? And why is the option of alternative algorithms emerging as the main event? The option for other alternative procedures should be replaced with required fluent use of standard algorithms. The emphasis should be on the standard algorithms. At the teachers' professional discretion alternative strategies and algorithms may be used to develop student understanding of why and how the standard algorithms work. Alternative strategies should be a sidelight, not the main event; they can obscure the use of the more efficient standard algorithms.

## IV. Comments about Requiring Students to "Understand" a Concept

We are extremely concerned about the fact that many of the standards are built around the word "Understand". For example, under the "Fraction" strand for grade 5, one standard states the following:

> *"Understand that multiplying a fraction by a/b means taking a parts of a decomposition of the fraction into b equal parts. For example, to multiply 2/3 * 4/5 = 8/15, one may decompose a whole of size 4/5 into 3 equal parts; each part has size 4/15. Two of these parts then make 8/15, so 2/3 * 4/5 = 8/15. (In general, a/b * p/q = ap/bq.) This standard includes multiplication of a whole number by a fraction, by writing the whole number as fraction with denominator 1."*

Another two in this same strand, state:

> *"Understand that the area of a rectangle with side lengths a/b and c/d is the product a/b ′ p/q. This extends the area formula for rectangles to fractional side lengths, and also allows products of fractions to be represented visually as areas of rectangles."*

> *"Calculate products of fractions, and quotients of unit fractions and nonzero whole numbers (with either as divisor), and solve word problems involving these operations. Represent these operations using equations, area models and length models."*

"Understand" as a leading verb results in an interpretation by different people for different purposes. To ask students to understand something, one teacher may expect them to show understanding by explaining while another may expect the student to show understanding by computing, comparing, or justifying. More precise leading verbs should be used to bring about the desired clarity and specificity that will convey the same meaning to all users.

An elementary school teacher reading these standards would likely ask "What is the standard the student must achieve?" How does a teacher ensure that the student "understands" the conceptual underpinning of fraction multiplication which is what the standards call for? The usual way is through testing, but then how is such understanding tested? It is obvious that the objective of the understanding is to be able to calculate products of fractions (and quotients of unit fractions and nonzero whole numbers) and solve word problems involving these operations. Then this is how such standards should be stated: "Students will be able to multiply fractions and apply that understanding in solving word problems." Such proficiency is sufficient and represents "understanding" in the sense that Bloom's taxonomy intends; that is, the student's understanding allows him or her to carry out a mathematical procedure and apply it in solving a problem. The standards as written assume that such proficiency is not sufficient and that it, in fact, is the same as rote memorization and that students will be lacking in mathematical ability.

The conceptual understanding stated in the standard is important to motivate the teaching of fraction multiplication. (Singapore Math for example, does this; ironically so did many of the textbooks from the 1950's and 1960's, an era that has been mischaracterized as one in which math procedures were taught in a rote fashion in isolation to their application). By all means, teachers should certainly use the rectangle area model to show how fraction multiplication works and to use to motivate the algorithm for it. Some students will retain this explanation; others will not. In the end, the understanding that students need is how to multiply fractions and apply it to solve word problems.

On page 3 of the standards document it states the following:

> *"The standards in this draft document define what students should understand and be able to do. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from."*

This requires a monumental leap. Would asking students to justify something be fair assessment if they only need to meet a standard asking them to understand? They would need to understand in order to *justify*, but justify is a higher order of Bloom's taxonomy. Understand and justify are thus very different. If a student is expected to justify, the standard should be written with *justify*, not understand. A standard should not call for a student to learn to do one thing and set up an assessment at a different level.

Testing for such understanding will likely amount to requiring students to "Show two ways to multiply 2/3 x 4/5", or "Use a rectangle model to show how to calculate 2/3 x 4/5". Requiring students to retain the first principles of the derivation of the algorithms is a burden, which likely will be manifest in students memorizing the derivations in rote fashion. In short, requiring such understanding will likely undermine the intent.

If standards are well written, they become the rudder of education, and effective methods of instruction and assessment will follow in its wake. If the standards are not well written, whether or not the content is there, assessment becomes the rudder, with instruction following in its wake —especially when the standards are written in such a way as to lend themselves to multiple interpretations, not of the content, but of what is expected of students. Well written standards lend themselves to good assessments. Poorly written standards lend themselves to poor or meaningless assessments. We have already experienced poor and meaningless assessments across the country.

The emphasis placed on understanding is particularly troubling not only because of its potential effect on assessment, but because such requirement aligns with some seriously deficient math programs that are being used in schools in the U.S. These programs emphasize understanding by asking students to provide two or three ways to solve a simple computation problem and asking them to explain their procedure in words or to draw pictures. Enforced understanding is ultimately ineffective. It fosters neither understanding nor the procedures to be mastered in order to solve problems. Standards should describe what the understanding will enable students to do procedurally and the types of problems they should be able to solve. As written, the standards in the current draft fall far short of that.

## V. Comments on the Standards

### A. Established Goal and Guiding Criteria

This draft of the K-12 Common Core State Standards falls short of meeting the established goal and does not meet the criteria for essential, rigorous, clear and specific, teachable and learnable, and measureable. Topic Mapping and comparison with the TIMSS benchmarks indicate the criteria for internationally benchmarked has not been met. No evidence for such benchmarking has been provided. At the end of the standards document is a section called "Sample of Works Consulted." In this section one item on the list is Mathematics documents from: Alberta, Canada; Belgium; China; Chinese Taipei; Denmark; England; Finland; Hong Kong; India; Ireland; Japan; Korea, New Zealand, Singapore; Victoria (British Columbia). Without specific information about each document consulted, others are not able to consult the same document.

### B. Areas of Strength

- The emphasis on place value is good
- The standards are a great improvement over the January draft in many ways, although it still falls short of being world class, thus not good enough for the students of this country.
- Statistics and Probability do not appear until grade 6. This allows a focus on more fundamental skills and concepts at the earlier grades.
- Math terminology is present. Consider using count instead of say and use numeral instead of number when appropriate.

- The format is improved overall.
- There is a sensible development toward multiple math pathways for workforce, college, and STEM rather than one size fits all. More work on this is still needed.
- Using a symbol to indicate standards for the development of algebraic thinking is a good way to make note of those standards. Add a little more information so it will be clear whether these standards should be given greater emphasis.

**C. Areas of Concern**

- The topical organization in high school is not acceptable.
- The high school standards will still not prepare non-STEM majors to qualify for entrance in most 4 year universities, since key topics in Algebra 2 are missing.
- Many standards are stated in terms of what students must "understand" rather than what they must be able to do, or what types of problems they must be able to solve.
- Many standards are verbose and proscriptive, and often repetitive and conflicting.
- Pacing is an issue; generally the standards are a year behind some high achieving countries, and by grade 5 possibly two years behind.
- There is not enough focus on fluency and practice
- Does not seem to promote the National Mathematics Advisory Panel's recommendations for algebra—standards do not appear to provide solid preparation for Algebra I in either grade 8 or 9.

**D. General Comments**

The K-8 Common Core standards are simultaneously over- and under- ambitious. Numerous computation skills are delayed or under emphasized, while, at the same time, children are expected to "understand" sophisticated ideas and principles that many teachers do not themselves understand. Although we applaud the ambitious goal of having children understand these things, this must be a long-term goal. Initial instruction on these topics must be in our colleges of education so that those entrusted with teaching our children will themselves - for the first time - develop Liping Ma's *Profound Understanding of Fundamental Mathematics* (PUFM). Until and unless elementary and middle school teachers develop PUFM, nothing good can come from standards that demand our children to "understand" sophisticated ideas that the adults in their lives have not yet mastered. U.S. textbooks neither teach, review nor test these things. Are classroom teachers expected to develop their own materials? How are our schools to make this change overnight, with the adoption of these standards?

It is unnecessary to have students explain the reasoning used for many of these standards. Knowing how to apply a skill, strategy, procedure, or algorithm is its own explanation. Asking for an explanation of reasoning will result in confusion, frustration, and ultimately rote learning of an explanation that most likely will not make sense to the student. Some standards are too granular in size and some of the standards are so broad stroke they miss all the relevant and necessary concepts needed for mastery.

Pacing and rigor are in need of improvement—for example, exponents/powers/roots are not explicitly covered until high school. STEM students should be covering much of the high school material in middle school to avoid boredom and enable acceleration through upper level math. Instead, the standards make STEM topics optional and only at higher grades. The standards should be written to encourage flexible pacing for varying abilities and interest. Lockstep instruction does not work for math after fifth grade (and for some students well before).

**E. Grades K-6**

There is insufficient time in a half-day kindergarten to cover the topics, and given the high variability of educational experiences of incoming Kindergarteners, many are simply not ready to be adding in different ways to 10.

The number sequence is too high in grade 1, Reduce to 30 counting. Rounding, on the other hand, is not introduced until grade 4 with expected knowledge. It should be introduced in grade 2 or 3. Introduce skip counting by 2,s & 5s in kindergarten, and skip counting by 3s in grade 2.

Ordinals are missing. We recommend ordinals 1st-20$^{th}$ be introduced in grades 1 and 2. Odd and even numbers are missing and should be introduced in grade 1 or no later than grade 2.

The comparison of multi-digit numbers is prescriptive. It should simply be as follows:

- 1st - compare & order numbers to 100
- 2nd - compare and order numbers to 1000,
- 3rd compare & order numbers to 1,000,000.

Place names are missing after thousands. Students should know ten thousands, hundred thousands, millions, billions, trillions. Place values less than one are not explicit. Students should know tenths, hundredths at least.

Properties of addition & multiplication are repetitive in their introduced grade level. The grade 1 standards do include many math strands that highly respected bodies (California, Massachusetts, Indiana, NCTM Focal Points) also include. However, there exists both language that softens the importance of these strands or neglects to emphasize key aspects of these strands. Understanding the properties of addition and subtraction as well as place value to 100 is demonstrated with these strands. Addition and subtraction of whole numbers is covered.

The language needs to be much more clear in other topic areas. First and foremost, grade 1 students need to develop fluency and automaticity with math facts involving addition and subtraction of whole numbers within 20. Students need automatic recall of these facts and this must start in grade 1. The standards mention mental math for adding and subtracting whole numbers within 20 and fluency is required for add/sub of whole numbers within 10. However, students need to have immediate recall of their facts within 20. Starting in grade 1 will enable students to accomplish this. The word "memorize" need not be something to avoid.

The following topics are missing in Kindergarten and grade 1:

- Clocks and time missing in K
- Coins and bills missing in K and1
- Calendars and dates missing for all.
- Making change missing

Grade 3 should introduce powers/exponents/roots. Grade 3 is a great place to introduce the concept of "square" along with multiplication and area models and "powers of ten" along with the decimal system. Grade 4 should include preparation for the Fundamental Theorem of Arithmetic (Unique-Prime-Factorization Theorem) which should be introduced in 5th grade.

Grade 5 mentions powers of 10 in the overview, but never makes evaluation of powers/exponents/roots explicit in the standards. At no grade level is there any mention of or skill development for simplifying fractions, finding prime factors, finding common denominators, finding least common multiples or denominators, and finding greatest common factors. Students are never asked to know what any of these are or to find any. Students are asked to find factor pairs and are only asked to recognize prime factors but not find them. Students are expected to write fractions in decimal notation for fractions with 2, 4, 5, 8, 10, and 100. The standards never develop the skill or knowledge of how to convert any fraction into a decimal.

Simplifying fractions, finding factor pairs, finding prime factors, finding common denominators, finding least common multiples or denominators, and finding greatest common factors are all important foundational skills critical for success in authentic algebra and beyond.

The standards are heavy with theory and pedagogy when it comes to fraction multiplication and division for grades 5 and 6. The standards for division of whole number by a unit fraction and division of unit fraction by a whole number again are too theoretical. They should be stated in terms of what the student should be able to do, rather than what the student must understand. Also, there is an omission of reciprocals. Reciprocals are easily introduced along with fraction multiplication and play a significant role in explaining fractional division later. Why the omission?

Geometry appears to be a problem in grades 5 and 6. In general, these standards are focused too much on analytic geometry. Shapes, angles, and congruence are concepts that can be taught in grade 5.

Exponents are not mentioned until grade 6 and then, only for area and volume. The next mention isn't until grade 8.

The terms "inequality" and "inequalities" are not mentioned until high school.

**F. Grade 7**

The topic mapping indicates standards should be added for Exponents in Grades 7 & 8 (see codes 22&23 on the topic mapping in the appendix). Computational mastery of these skills is needed in preparation for authentic algebra and many jobs.

The terminology and evaluation of "exponent," "square," "power," "scientific notation" and "root" are mentioned only in passing or completely omitted until high school. Explicit explanation of the these terms is essential for students, including rules for multiplication by powers of ten and rules for evaluating exponents, especially fractional, negative, or multiplicative exponents. These topics are often introduced in grade 6.

Relative to expectations in the A+ countries and states with top-rated standards, the standards for grade 7 students are somewhat less advanced. Although this may be appropriate for students who need extra time, many if not most U.S. students should be prepared to begin study of higher mathematics (e.g., algebra I) in grade 8 or earlier. The standards should indicate how this faster pace might be accomplished to provide guidance for teachers, textbook authors and others involved in planning instruction for students working at a more internationally competitive pace.

In any case, mastery of arithmetic should be completed by at least the end of grade 7. The standards come fairly close to achieving this modest goal, but several improvements are still needed:

1) It should be stated clearly and unequivocally that students must be fluent in using standard algorithms to add, subtract, multiply, and divide integers, fractions, mixed numbers, and terminating decimals. Students should be able to handle decimal divisors with hundredths digits and fractions with denominators such as 7 or 11. Revisions should be made to ensure that all these basic skills are mastered by at least the end of grade 7. In the current draft it is unclear whether students must achieve proficiency in actually performing calculations or if they only need to understand the concept. Revisions should make it clear that both are necessary.

2) Students must be able to convert fractions to decimals and percents, and must be able to convert terminating decimals into reduced fractions. They must also be able to compare and order numbers that are presented in a variety of forms

(e.g., fraction, decimal, percent). These topics are not currently addressed.

3) Proficiency in manipulating expressions that involve whole-number exponents should be achieved at least by the end of grade 7. This includes fluency using exponent rules to multiply, divide, and simplify expressions with signed, whole number exponents. In the current draft, it is unclear whether students are expected to master this.

Other standards topics reflect a moderate delay in expectations. Although proportional relationships are covered relatively thoroughly in grade 7, the study of other linear relationships is postponed until grade 8. Both Massachusetts and California standards include significant work with (non-proportional) linear equations in grade 7. Fluency in applying the order of operations must be achieved at least by the end of grade 7, but the standards fail to make this clear. Inequalities, which are covered in grade 7 by other standards documents, are missing in the grade 7 standards. Multiplication, division, and taking roots of monomials is also absent. Scientific notation is covered in grade 7 in both Massachusetts and California; the Common Core standards delay this topic until grade 8.

Coverage of geometry topics is a relative strength of the grade 7 standards, due largely to the grade 6 focus on computing area, volume, and surface area of 2- and 3-dimensional figures. Notable exceptions are the delay in studying the Pythagorean theorem until grade 8 and the lack of standards relating to constructions with straight-edge and compass.

Statistics and probability receive more attention than is afforded by other top-rated standards documents. Reduction in the scope of coverage of these topics would make room for essential topics that are covered later or in less depth.

**G. Grade 8**

The most troublesome areas not covered or in need of better coverage in the standards are the following: multiples (grade 6, needs better coverage), exponents, perfect squares, estimating and approximating decimals, conversion between percents and decimals, exponents and expressions, evaluating an expression or equation given a value for a variable (can be more explicit) solving linear equations with inequalities, and language used to define angles (i.e., acute, obtuse, right).., Of great concern is that skills like exponents, multiples, weak coverage of percents, decimals and fractions and solving inequalities are significant skills that are lacking or not covered.

Other skills not covered or lacking in the standards but not as critical as those mentioned above are as follows: angle bisection, chance which is hardly covered and not clear, mode, the ability to explain the misinterpretation of a set of data, pie charts, comparing and matching different representations of data, patterns, formulas explicitly defined for area and volume, spheres, and rotational symmetry. Mode is surprising since other topics are covered in the standards like mean and median.

The grade 8 standards are limited to three "critical" areas. There is no research base cited for limiting the grade 8 standards in this manner. Moreover, important topics from the NCTM Focal Points, Massachusetts's standards, Achieve, and TIMSS topics are missing with no justification. It is important to note that the March draft is a significant improvement over earlier drafts in that the first topic–solving linear equations and systems of linear equations–is a solid introduction into algebra. Prior drafts did not separately address linear equations and systems of equations. They were rolled into the functions topic without adequate attention.

A major flaw of the grade 8 standards is they don't appear to follow logically from grade 7 and flow into high school. They lack articulation between grade 7, 8 and high school. The lack of articulation shows up in the topics that are eliminated from grade 8, including measurement (Massachusetts, TIMSS, NCTM Focal Points, NMAP), exponents, roots, and absolute value (Achieve, TIMSS, Massachusetts), prime numbers, prime factorization (Massachusetts, TIMSS), and fractions.

Despite the strong emphasis in NMAP on the need for procedural and conceptual fluency in fractions, proportions, ratios, scale factors, and formal problem solving using fractions the standards never mentions "fractions" in grade 8.

Grade 8 appears to be in some ways, an island onto itself and it isn't clear how students will get from grade 7 to 8. Requiring students "solve and explain word problems leading to two linear equations in two variables" is a great goal for grade 8, but may not be reasonable given the lack of foundation students will have in fractions, proportion, ratio, etc. in earlier grades.

The major flaw, identified by Wurman and Stotsky, in the September and January drafts, remains. The CCSSI "jeopardizes the teaching of Algebra 1 in grade 8." The current draft contains 3 topics in algebra (linear equations in one variable, linear equations in two variables, and systems of linear equations). There remains "little to no exposure to coordinate planes, the law of exponents, roots, irrational numbers, or functions, except for simple linear ones."

The inadequate development of algebraic reasoning in K-7 or K-8 will not have students algebra ready in grades 8 or 9

## H. High School

### 1. General High School Comments

These standards are inconsistent with current college entrance requirements. What will become of students who will take the ACT and SAT exams in a few years, or the expectation of College Algebra? There is a plethora of probability and statistics repeatedly throughout these standards at the cost of developing rigorous, authentic algebra. Students won't have the foundation for the study of College Algebra with these standards in place.

The organization of the High School Standards is severely lacking. No guidance is provided for teachers that would enable them to assist students in building their mathematical knowledge. Topics are scattered with few connections noted.

There's mention of inverse functions in standards, but no development of logarithmic functions as inverses of exponentials. All high school algebra students should be able to algebraically solve simple exponential equations like $2^x = 9$.

Almost all standards involving complex numbers are marked STEM. Since they are currently assessed on the ACT exam should they be for all or STEM only.

2. Numbers and Quantity

In order to be consistent with grade 8, The Number System (1), the Real Number System (3) should be stated: "Understand that sums, differences, products and quotients of rational numbers are rational." An example like (2.456)/(0.34) is needed here so that students are expected to demonstrate their ability to apply either the standard division algorithm or division of fractions and write the quotient in rational form. This is not necessarily required in Quantities (standard 4).

3. The Complex Number System

The introduction leads one to believe complex numbers will be fully developed for all students. Most standards in this section are marked STEM, which, means all students would not be taught this material. All algebra 2 students need to learn this material or they will not be prepared for pre-calculus level study. Complex numbers are assessed on the ACT exam because they are now considered basic algebra.

Standards 2, 3, 5, 7, 8, 9, and 11 are Algebra 2 and should not be marked STEM. The remaining standards in this section are typical pre-calculus/trig content.

Standard 2: This should not be restricted to STEM students. Without this basic understanding, all of the standards referring to "parameter changes" on quadratic functions is meaningless because students will have no understanding of the nature of the roots for graphs not intersecting the x-axis.

Standard 6: If all students are expected to add, subtract and multiply complex numbers, why wouldn't they be expected to use them for something?

Standard 7 should also include the sum and product rule for quadratic equations, which beautifully demonstrates why the solutions to a quadratic equation with real coefficients can indeed be complex conjugates.

4.Vector Quantities and Matrices

Standards 6, 8, 9, 13, 15 and 16 should not be marked as STEM. All students should understand matrix operations and their applications well enough to set-up and algebraically solve a system of two linear equations. Standard 13 is a duplicate of 6 (which is stated much better in 6). Standards 8 and 15 should be combined.

5. High School Algebra

The ninth paragraph of the introduction explains exactly why the STEM demarcation should not be on most standards in Complex Number: "Some equations have no solutions in a given number system, stimulating the extension of that system." We question whether this is only for STEM students, and if so, why?

6. Seeing Structure in Expressions

This would be an excellent place to include the study of Pascal's triangle, especially since the Binomial Theorem is marked STEM in the following section. If students expanded binomial expressions of the form $(x + y)^n$ using Pascal's triangle, they would be better prepared to understand applications of them in constructing efficient probability models for some of the examples included on p. 56-57.

7.Arithmetic with Polynomials and Rational Expressions

The zero-product property should be included in standard 7. The rational root theorem should also be included in this section.

Standard 6 should include the standard algorithm for long division of polynomials. Eliminate standard 9 and 11 because they would be included in the corrected form of standard 6 for all students, not just STEM students. The division algorithm should not be restricted to only divisors of the form x – a at the high school level. All algebra students should be able to divide a polynomial by another polynomial of equal or lesser degree. The restriction to "x - a" is needed here only because the standard algorithm for division has been avoided previously. Standard 10 should not be marked STEM. This is where the understanding of domain and range becomes meaningful for students. The study of direct and inverse variation, which should be included in Creating Equations (3) naturally leads to linear graphs, but also nonlinear graphs that have asymptotes.

8. Creating Equations That Describe Numbers or Relationships

Standard 3 should naturally include direct variation, joint variation and inverse variation. The latter two would require the student to understand why vertical and horizontal asymptotes are present in the graphs. An example of this is given with standard 8 in Interpreting Functions, but inverse variation is an important topic which should not be overlooked in this section.

Standard 4: The elementary understanding of inverse operations demonstrated in this standard should have been fully developed in the grade 8. There is an example which may appear to some to be quite challenging, but in reality it merely demonstrates the pre-algebra skill of dividing both sides of the equation by 2ax. "Rearranging formulas" in this sense is pre-algebra.

9. Reasoning with Equations and Inequalities

Standard 2 includes complex numbers for all students unless you note that q is a nonnegative rational number. The standard is misleading the way it is currently written. It would be better to state "Complete the square to transform quadratic equations and develop the quadratic formula." All students should be able to do.

Standard 9 and 20 should not be marked STEM. All students should be able to solve simple exponential functions like $3^x = 10$ algebraically. Without that, the study of pH or exponential growth in science classes will be very difficult.

Standard 12 should include the fact that the quadratic formula is derived by the method of "completing the square." The standard is stated as "Solve quadratic equations in one variable" which includes complex solutions – whether that was intended or not. All students need the study of complex numbers. How are teachers expected to define the number e, in standard 19, at this level? As the base of the natural logarithmic function? It hasn't been defined yet.

10. Interpreting Functions

Standard 5 refers to "long-run behavior" and "end-behavior" is mentioned earlier. As used, are these to mean the same thing? If so, use consistent terminology to mean the same thing.

Standard 10: Students must know how parameter changes affect graphs without technology. Students routinely enter the wrong equations into their calculators or computers by accident. Without basic knowledge, they will have no hope of catching entry errors These graphs rarely fit perfectly into a given window. Students must know how to make appropriate adjustments in order to see the graph.

11. Building Functions

Standard 4, 10, 11, and 12 should not be marked STEM. This is algebra 2 content. All students should be able to solve simple exponential functions of the general form $a^x = b$.

12. Statistics and Probability

There are too many standards on probability and statistics. This material is not required in high school to be successful in college level statistics courses if students have a firm foundation in algebra. It's not assessed at this level on the ACT exam.

13. Geometry

The high school geometry standards cover a good deal of the concepts that students should learn in a standard geometry course. It is not evident from reading the standards that the standards require that students be able to construct deductive proofs of geometric conjectures. We applaud the standards for requiring students to be able to construct proofs of some key theorems including the Pythagorean Theorem. But there do not appear to be standards requiring students to conduct deductive proofs beyond key theorems. As such, geometry courses will be watered down, focusing on problem solving by applying formulae and general principles, but without developing the skills to set up a series of statements that systematically lead from given conditions to a specific conclusion. (Example: Standard 5 under Geometry, Circles states "Determine the arc lengths and the areas of sectors of circles, using proportions."

Typical courses in geometry are lacking in proof-based problems; instead, they contain many problems in which missing angles or segments are indicated as algebraic expressions. For example, two congruent segments in a geometric figure may be labeled x + 2 and 2x + 6; the student is asked to find the length of the segments. While this involves some deductive reasoning to conclude that the two segments of interest are congruent, such exercises do not fully develop the skills necessary to develop a logical series of statements that constitute proof. Such exercises amount to nothing more than identifying generally obvious congruent or similar parts of figures and solving algebraic equations. The standards as written will allow such treatments to continue.

Also, typical of most textbooks today, the geometry standards place much emphasis on transformations, analytic geometric concepts and skills, and trigonometry. Transformations are used in these standards in applying principles of congruence and similarity. There is a lack of standards requiring students to be able to apply the principles of proportions in similar figures. Rather, the standards for similarities use terms such as "dilations" and "scale factors". While correct and embodying the concepts of proportions, the use of proportions is noticeably absent in these standards.

Standard 6 of geometric measurement and dimension states: "For a pyramid or a cone, give a heuristic argument to show why its volume is one-third of its height times the area of its base." This standard could be interpreted to use the approach used in grade 5 in which a cone shaped cup of a given height and diameter of its circular base is used to measure how many cupfuls of water will fill a cylinder with the same height and circular diameter. It is not clear that the key geometric principles will be used (identification of the three congruent pyramids in a right triangular prism, and application of Cavalieri's principle to extend the volume of a triangular pyramid to other types of pyramids and that of a cone).

## VI. Conclusions and Recommendations

The presentation of these standards is significantly different than standards have been presented and applied by states up to this point. As such, the practical approach may be to see how it will work before states are required to use them. (Diane Ravitch has said these new standards should be tried on a very small scale first before launching them nationwide.) And at the same time, using solid research methodology, compare results with results from the implementation of the Massachusetts, California, and Indiana standards. Let's put something in place that has proven itself, rather than risk, not only untold resources, but the development of our students mathematical achievement. Far better to take the time now to field test these standards than to unleash them on our students.

On page 7, the standards document says:

> *"One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding."*

This would be a good reason for having several consortiums of states developing (preferably selecting a ready-made set and making modifications) sets of standards. In addition to giving states some choice (greater than 15%) then we can see which ones seem to work the best.

We recommend that the CCSSI do the following:

1) Field test the common core state standards on a limited basis for a minimum of one year prior to state-wide use across the country;
2) actively support, promote, and help states establish consortiums and develop sets of standards; and
3) declare the math standards from Massachusetts, California, and Indiana acceptable alternatives to the common core state standards so that states will have choices.

As the CCSSI goes about revising the standards, we recommend they give attention to and address all specific issues presented in this report. We feel it especially important that the CCSSI give priority to..

1) Replace "understand" with more precise measurable verbs that will clarify the desired proficiency level;

2) remove noted ambiguities regarding standard algorithms by removing strategy related standards and clearly requiring fluent use of standard algorithms to add, subtract, multiply, and divide whole numbers, integers, fractions, and decimals;
3) specifically include standards that develop skills related to simplifying fractions, finding factor pairs, finding prime factors, finding common denominators, finding least common multiples or denominators, and finding greatest common factors to provide students with the necessary foundation for success in authentic algebra and beyond;
4) ensure the geometry standards provide a fundamental foundation in synthetic geometry requiring students to construct deductive proofs; and
5) ensure the standards fulfill the minimum mathematics requirement for 4-year universities and prepare students to take subsequent courses in mathematics without the need for remediation.

We find the March draft of the Common Core Standards for Mathematics K-12 to be a considerable improvement over the January draft. We also find the standards as currently written to be better than the standards of many states. These standards are not yet on an equal level or better than the current state standards for Massachusetts, California, and Indiana and as such are not world class. Anything less than world class standards for the students of this country is not acceptable. We hope the Common Core State Standards Initiative will bring these standards up to the level of excellence to qualify as world class.

Appendix

Topic Mapping

TIMSS comparison

Who We Are

## Common Core State Standards March Draft K-12
## Mathematics Topics at Each Grade

| Legend |
|---|
| Match A+ |
| Missing |
| Extra |

| Code | Topic | GdK | Gd1 | Gd2 | Gd3 | Gd4 | Gd5 | Gd6 | Gd7 | Gd8 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Whole Number Meaning | 15 | 4 | 6 | 4 | 2 | | 1 | | 1 |
| 2 | Whole Number Operations | 10 | 7 | 7 | 5 | 5 | 3 | 1 | 1 | |
| 3 | Measurement and Units | 2 | 5 | 9 | 3 | 1 | 10 | 1 | 2 | |
| 4 | Common Fractions | | 2 | 1 | 7 | 9 | 5 | 1 | 1 | |
| 5 | Equations and Formulas | 6 | 3 | 3 | 4 | 2 | 1 | 9 | 6 | 5 |
| 6 | Data Representation & Analysis (Graphing) | | 1 | 2 | 2 | 1 | 1 | 4 | | 2 |
| 7 | 2-D: Basics | 2 | | | | 7 | | | 1 | 1 |
| 8 | Polygons & Circles | 6 | 5 | 5 | 3 | 2 | 2 | | 1 | 4 |
| 9 | Perimeter, Area & Volume | | | | 5 | 3 | 3 | 7 | 1 | |
| 10 | Rounding & Significant Figures | | | | | 1 | 1 | | | |
| 11 | Estimating Computations (incl. mental math) | | 2 | 2 | 2 | 3 | | 1 | | |
| 12 | Properties of Whole Number Operations | 1 | 7 | 3 | 8 | 4 | 2 | 1 | | |
| 35 | Estimating Quantity and Size | | | | | | | | | |
| 13 | Decimal Fractions | | | | | 3 | 6 | | | |
| 14 | Relationship of Common & Decimal Fractions | | | | | 2 | 8 | | | |
| 15 | Properties of Common & Decimal Fractions | | | | | | 5 | 3 | | 1 |
| 16 | Percentages | | | | | | | | 3 | |
| 17 | Proportionality Concepts | | | | | | | 4 | 4 | 1 |
| 18 | Proportionality Problems | | | | | | | 2 | 2 | |
| 19 | 2-D Coordinate Geometry | | | | | | 2 | 2 | 3 | 4 |
| 38 | Transformations | 1 | | | | 2 | | | 1 | 1 |
| 20 | Negative Numbers, Integers & Their Properties | | | | | | | 2 | 2 | |
| 21 | Number Theory (primes & factorization) | | | | | 1 | | | 1 | |
| 22 | Exponents, Roots & Radicals | | | | | | | 1 | | |
| 23 | Exponents & Orders of Magnitude | | | | | | | | | |
| 36 | Measurement Estimation and Errors | | | | | | | | 1 | |
| 24 | Constructions w/ Straightedge & Compass | | | | | | | | | 1 |
| 25 | 3-D Geometry | 3 | 1 | 1 | | | | | | 2 |
| 26 | Congruence & Similarity | | | | | | | | 5 | 4 |
| 27 | Rational Numbers & Their Properties | | | | | | | 2 | 3 | 1 |
| 28 | Patterns, Relations & Functions | | | | | | | | | 7 |
| 29 | Slope & Trigonometry | | | | | | | | | 4 |
| 30 | Uncertainty & Probability | | | | | | | 3 | 7 | 5 |
| 31 | Real Numbers | | | | | | | | 1 | 2 |
| 32 | Binary Arithmetic and/or Other Number Bases | | | | | | | | | |
| 33 | Complex Number and Their Properties | | | | | | | | | |
| 34 | Counting | | | | | | | | | |
| 37 | Vectors | | | | | | | | | |
| 39 | Linear Interpolation and Extrapolation | | | | | | | | | |
| 40 | Infinite Processes | | | | | | | | | |
| 41 | Change (Growth and Decay, Differentiation) | | | | | | | | | |
| 42 | Validation and Justification | | | | | | | | | 1 |
| 43 | Structuring and Abstracting | | | | | | | | | |
| 44 | Other Content | | | | | | | | | |
| 97 | Vague | 1 | | | 1 | | | | | |
| 98 | Advocates Pedagogy | 7 | 6 | 5 | 5 | 4 | 2 | | 5 | 5 |
| 99 | Unmapped | | 1 | | 1 | | | | | 1 |
| | Count | 9 | 10 | 10 | 10 | 16 | 13 | 17 | 19 | 18 |

## FIGURE 1

*A+ Composite: Mathematics topics intended at each grade by at least two-thirds of A+ countries.*

*Note that topics are introduced and sustained in a coherent fashion, producing a clear upper-triangular structure.*

| TOPIC GRADE: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Whole Number Meaning | ■ | ■ | ■ | ▒ | ▒ | | | |
| Whole Number Operations | ■ | ■ | ■ | ■ | ▒ | | | |
| Measurement Units | □ | ■ | ■ | ■ | ■ | ■ | ▒ | |
| Common Fractions | | | □ | ■ | ■ | ▒ | | |
| Equations & Formulas | | | □ | ▒ | ▒ | ▒ | ■ | ■ |
| Data Representation & Analysis | | | □ | □ | ▒ | ▒ | | □ |
| 2-D Geometry: Basics | | | □ | ▒ | ▒ | ▒ | ■ | ■ |
| Polygons & Circles | | | | ▒ | ▒ | ▒ | ■ | ■ |
| Perimeter, Area & Volume | | | | ▒ | ▒ | ▒ | ▒ | □ |
| Rounding & Significant Figures | | | | ▒ | ▒ | | | |
| Estimating Computations | | | | ▒ | ▒ | ▒ | | |
| Properties of Whole Number Operations | | | | □ | ▒ | | | |
| Estimating Quantity & Size | | | | □ | □ | | | |
| Decimal Fractions | | | | ▒ | ■ | ▒ | | |
| Relationship of Common & Decimal Fractions | | | | ▒ | ■ | ▒ | | |
| Properties of Common & Decimal Fractions | | | | | ▒ | ▒ | | |
| Percentages | | | | | ▒ | ▒ | | |
| Proportionality Concepts | | | | | ▒ | ▒ | ▒ | □ |
| Proportionality Problems | | | | | ▒ | ▒ | ■ | ■ |
| 2-D Coordinate Geometry | | | | | □ | □ | ▒ | ▒ |
| Geometry: Transformations | | | | | | ▒ | ▒ | ▒ |
| Negative Numbers, Integers & Their Properties | | | | | | □ | ▒ | |
| Number Theory | | | | | | | ▒ | □ |
| Exponents, Roots & Radicals | | | | | | | ▒ | ▒ |
| Exponents & Orders of Magnitude | | | | | | | □ | □ |
| Measurement Estimation & Errors | | | | | | | □ | |
| Constructions w/ Straightedge & Compass | | | | | | | ■ | □ |
| 3-D Geometry | | | | | | | ▒ | ■ |
| Congruence & Similarity | | | | | | | | ■ |
| Rational Numbers & Their Properties | | | | | | | | □ |
| Patterns, Relations & Functions | | | | | | | | □ |
| Slope & Trigonometry | | | | | | | | □ |
| Number of topics covered by at least 67% of the A+ countries | 3 | 3 | 7 | 15 | 20 | 17 | 16 | 18 |
| Number of additional topics intended by A+ countries to complete a typical curriculum at each grade level | 2 | 6 | 5 | 1 | 1 | 3 | 6 | 3 |

□ – intended by 67% of the A+ countries ▒ – intended by 83% of the A+ countries ■ – intended by 100% of the A+ countries

A Coherent Curriculum The Case of Mathematics By William Schmidt, Richard Houang, and Leland Cogan
AMERICAN EDUCATOR SUMMER 2002

# TIMSS 4th and 8th Grade Comparison

## 4th Grade

The Common Core in 4th grade and below hits many topics appropriately, but there are many outages that should be rectified and aligned to the 4th grade TIMSS. The Common Core does not expect students to write words to express numbers or read numbers from words. This is a basic skill that should be included in the Common Core. Overall, there is a very weak coverage of factors and multiples are not covered in the Common Core. There must be better coverage of factors and multiples. Both factors and multiples are critical foundations for fractions. There should be coverage of weight and temperature in the Common Core and measurement could be covered better. The Common Core does not introduce decimal operation until fifth grade. The TIMSS expects students to add and subtract decimals and solve problems that involve adding and subtracting decimals. It would be appropriate to include these skills in 4th grade. Estimation should be included throughout the Common Core document. In the Common Core, students are expected to measure lengths but not expected to estimate. Using coordinate systems to locate points in a plane is not expected until 5th grade; it may be appropriate to align this skill to 4th grade. The ability to draw in draw reflections and rotations of a figure is a 7th grade skill in the Common Core; this should become a 4th grade skill.

In the Common Core, there has been a significant reduction of topics in using data to answer and solve problems. This may be appropriate to focus the content on fewer topics. However, the following representations of data are missing from the Common Core and should be included: tables and pie charts. Since data is covered some in the Common Core, the Common Core should be more explicit in comparing data, organizing data and using the data to answer questions and solve problems.

The following are not covered in the Common Core below 4th grade and given the attempt to be more focused, the exclusion of these topics may be considered to be appropriate. It could considered appropriate that patterns, proportions, relationships between two-dimensional and three dimensional objects and volume are not covered.

# 8$^{th}$ Grade

Comparison of the Common Core to the TIMSS 8$^{th}$ grade benchmarks is troublesome. This clearly points out some major topics that were not covered at all that are necessary for Algebra I and shows that by 8$^{th}$ grade that the Common Core is lagging international expectations.

The most troublesome areas that are not covered or need better coverage in the Common Core are the following: multiples (6$^{th}$ grade, needs better coverage), exponents, perfect squares, estimating and approximating decimals, conversion between percents and decimals, exponents and expressions, evaluating an expression or equation given a value for a variable (can be more explicit) solving linear equations with inequalities, language used to define angles (acute, obtuse, etc.), and chance. Of great concern is that skills like exponents, multiples, weak coverage of percents, decimals and fractions and solving inequalities are significant skills that are lacking or not covered.

Other skills that are not covered or lacking in the Common Core but are not as critical as those mentioned above are: angle bisection, chance which is hardly covered and is not clear, mode, the ability to explain the misinterpretation of a set of data, pie charts, comparing and matching different representations of data, patterns, formulas explicitly defined for area and volume, spheres, and rotational symmetry. Mode is surprising since other topics are covered in the Common Core like mean and median.

# $4^{th}$ Grade TIMSS Comparison

| Numbers: Whole Numbers | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Represent whole numbers using words, diagrams, or symbols. | No, not explicit. It should address how to write whole numbers using words. |
| 2. Demonstrate knowledge of place value, including recognizing and writing numbers in expanded form. | Yes. |
| 3. Compare and order whole numbers. | Grade 4. |
| 4. Know the four operations ( +, –, ×, ÷) and compute with whole numbers. | Grade 3 and 4 |
| 5. Recognize multiples and factors of numbers; read weight and temperature scales marked in multiples. | Factors - some in grades 3 and 4. Multiples are not in the standards. Weight and temperature are not covered. |
| 6. Estimate computations by approximating the numbers involved. | Grade 4. |
| 7. Solve problems, including those set in real life contexts (for example, measurement and money problems). | Yes, but could be more explicit for solving measurement problems. More is covered in Grade 5. Yes, money is in Grade 3. |
| 8. Solve problems involving proportions | Grade 7 |

| Number: Fractions and Decimals | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Recognize fractions as parts of unit wholes, parts of a collection, locations on number lines, and divisions of whole numbers. | Yes. Grade 3 and Grade 4. |
| 2. Represent fractions using words, numbers, or models. | Yes numbers and models. No words. |
| 3. Identify equivalent fractions; compare and order fractions. | Grade 4 |
| 4. Add and subtract simple fractions. | Grade 4 |
| 5. Show understanding of decimal place value including recognizing and writing decimals using words and numbers. | Grade 4 - place value. No to words. |
| 6. Add and subtract decimals. | Grade 5 |
| 7. Solve problems involving simple fractions or decimals. | Grade 4 - Fractions, Grade 5- Decimals |

Note: Fourth-grade fractions items will involve denominators of 2, 3, 4, 5, 8, or 10. Fourth-grade decimals items will involve decimals to tenths and/or hundredths.

# $4^{th}$ Grade TIMSS Comparison

| Number: Number Sentences with Whole Numbers | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Find the missing number or operation in a number sentence (e.g., if 17 + __ = 29, what number would go in the blank to make the number sentence true?). | Yes. Earlier Grades. |
| 2. Model simple situations involving unknowns with expressions or number sentences. | Yes. |

| Number: Patterns and Relationships | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Extend patterns and find missing terms in them. | No |
| 2. Describe relationships between adjacent terms in a sequence or between the sequence number of the term and the term. | No |
| 3. Generate pairs of whole numbers following a given rule (e.g., multiply the first number by 3 and add 2 to get the second number). | No |
| 4. Write or select a rule for a relationship given some pairs of whole numbers satisfying the relationship. | No |

| Geometric Shapes: Lines and Angle | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Measure and estimate lengths. | Yes- for measure in earlier grades. No for estimate. |
| 2. Identify and draw parallel and perpendicular lines. | Grade 4 |
| 3. Compare angles by size and draw angles (e.g., a right angle, angles larger or smaller than a right angle). | Grade 4 |

# 4th Grade TIMSS Comparison

| Geometry Shapes and Measures: Two- and Three-dimensional Shapes | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Identify common geometric shapes. | Yes- covered in earlier grades. |
| 2. Know, describe, and use elementary properties of geometric figures. | Yes – covered in earlier grades. |
| 3. Classify and compare geometric figures, (e.g., by shape, size or properties). | Yes – earlier grades |
| 4. Recognize relationships between three-dimensional shapes and their two-dimensional representations. | No. |
| 5. Calculate areas and perimeters of squares and rectangles of given dimensions | Grade 3 and 4 |
| 6. Determine and estimate areas and volumes (e.g., by covering with a given shape or by recognizing that area is conserved). | Grade 3 and 4 - area, Grade 5 - volume |

| Geometric Shapes and Measures: Location and Movement | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Use informal coordinate systems to locate points in a plane. | Grade 5 |
| 2. Recognize and draw figures with line symmetry. | Grade 4 |
| 3. Recognize and draw reflections and rotations of figures. | Grade 7 |

| Data Display: Reading and Interpreting | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Read data from tables, pictographs, bar graphs, and pie charts. | Bar Graphs – Grade 3 and 4, Picture Graphs – Grade 4. Tables and pie charts - No |
| 2. Compare information from related data sets (e.g., given data or representations of data on the favorite flavors of ice cream in four or more classes, identify the class with chocolate as the most popular flavor). | Yes- Grade 1 and on. |
| 3. Use information from data displays to answer questions that go beyond directly reading the data displayed (e.g., combine data, perform computations based on the data, draw conclusions, and make predictions). | Not explicit. |

# 4th Grade TIMSS Comparison

| Data Display: Organizing and Representing | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Compare and match different representations of the same data. | No. |
| 2. Organize and display data using tables, pictographs, and bar graphs. | Picture graph- Grade 2, Bar Graph – Grade 2 and 3, Table – No. |

# 8th Grade TIMSS Comparison

| Number: Ratio, Proportion and Percent | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Identify and find equivalent ratios; express ratios. | Yes. Grade 6 and Grade 7 |
| 2. Divide a quantity in a given ratio. | Yes. Grade 6 and Grade 7 |
| 3. Convert between percents and fractions or decimals. | Yes – for fractions, Grade 7 - No for decimals. |
| 4. Solve problems involving percents and proportions. | Yes. Grade 7 – percents. Yes – proportions (Grade 6 and 7). |

| Algebra: Patterns | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Extend numeric, algebraic, and geometric patterns or sequences using numbers, words, symbols, or diagrams; find missing terms. | No |
| 2. Generalize pattern relationships in a sequence, or between adjacent terms, or between the sequence number of the term and the term, using numbers, words, or algebraic expressions. | No |

| Algebra: Algebraic Expressions | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Find sums, products, and powers of expressions containing variables. | Yes – sums and products. No – powers of expressions. |
| 2. Evaluate expressions for given numeric values of the variable(s). | Yes – Grade 6. Could be more explicit. |
| 3. Simplify or compare algebraic expressions to determine equivalence. | Yes - Grade 6 |
| 4. Model situations using expressions. | Yes - Grade 6 |

# $8^{th}$ Grade TIMSS Comparison

| Number: Whole Numbers | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Demonstrate knowledge of place value and of the four operations. | Yes |
| 2. Find and use multiples or factors of numbers, read scales, and identify prime numbers. | Yes – factors and prime numbers. Multiples in 6th grade, but could be covered better. Yes - scale, Grade 7 |
| 3. Use the principles of commutativity, associativity, and distributivity. | Yes. Distributive property starts in Grade 3 (seems early) |
| 4. Evaluate powers of numbers, and square roots of perfect squares to 144. | Very limited exposure to powers (6th grade). Perfect squares are not addressed. |
| 5. Solve problems by computing, estimating, or approximating. | Yes |

| Number: Fractions and Decimals | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Compare and order fractions and decimals. | Yes |
| 2. Demonstrate knowledge of place value for decimals. | Yes |
| 3. Represent decimals and fractions and operations with decimals and fractions using models (e.g., number lines); identify and use such representations. | Yes |
| 4. Recognize and write equivalent fractions. | Yes |
| 5. Convert between fractions and decimals. | Yes |
| 6. Compute with fractions and decimals. | Yes |
| 7. Solve problems by computing, estimating, and approximating. | Yes for fractions. No for estimating and approximating for decimals. |

| Number: Integers | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Represent, compare, order, and compute with integers. | Yes – Grade 6 and 7 |
| 2. Solve problems using integers. | Yes – Grade 7 |

# $8^{th}$ Grade TIMSS Comparison

| Algebra: Equations/Formula and Functions | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Evaluate equations/formulas given values of the variables. | Not explicit |
| 2. Indicate whether a value (or values) satisfies a given equation/formula. | Yes – Grade 6, could be more explicit. |
| 3. Solve simple linear equations and inequalities, and simultaneous (two variables) equations. | Linear Equation - Grade 8, Inequality - No, Simultaneous equations – Yes. |
| 4. Recognize and write linear equations, inequalities, simultaneous equations, or functions that model given situations. | Linear Equation, Simultaneous Equations, Functions - Grade 8, Inequalities – No. |
| 5. Recognize and generate equivalent representations of functions as ordered pairs, tables, graphs, or words. | Yes |
| 6. Solve problems using equations/formulas and functions. | Yes |

| Geometry: Geometric Shapes | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Classify angles as acute, right, straight, obtuse, and reflex (more than 180˚); draw such angles. | No. This vocabulary is missing. |
| 2. Know and use the relationships for angles at a point, angles on a line, vertically opposite angles, angles associated with a transversal cutting parallel lines, angle bisection, and perpendicularity. | Yes -Grade 8.<br>No – angle bisection |
| 3. Recall and use geometric properties of geometric shapes: triangles, quadrilaterals, and other common polygons. | Yes |
| 4. Construct or draw triangles and rectangles of given dimensions. | Yes |
| 5. Identify congruent triangles, quadrilaterals and their corresponding measures. | Yes – Grade 7. Could be more explicit. |
| 6. Identify similar triangles and recall their properties. | Yes - Grade 7, more Grade 8. |
| 7. Recognize relationships between three-dimensional shapes and their two-dimensional representations, (e.g., nets or two-dimensional views of three-dimensional objects). | Yes - Grade 6 |
| 8. Use Pythagorean theorem (not proof) to solve problems. | Grade 8 |
| 9. Apply geometric properties to solve problems. | Yes |

# $8^{th}$ Grade TIMSS Comparison

| Geometry: Geometric Measurement | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Measure, draw, and estimate the size of given angles. | Yes |
| 2. Measure, draw, and estimate the length of lines, perimeters, areas and volumes. | Yes |
| 3. Select and use appropriate measurement formulas for perimeters, circumferences, areas of circles, surface areas, and volumes. | Yes. Surface area is spelled out for cubes, prisms, and pyramids. Formulas are not explicitly spelled out except for rectangles and prisms. Spheres are not mentioned. |
| 4. Find measures of irregular or compound areas (e.g., by covering with grids or dissecting and rearranging pieces). | Yes |

| Geometry: Location and Movement | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Use ordered pairs, equations, intercepts, intersections, and gradient to locate points and lines in the Cartesian plane. | Yes |
| 2. Recognize and use line and rotational symmetry for two-dimensional shapes, e.g. to draw symmetrical figures. | Yes – line symmetry. No – rotational symmetry. |
| 3. Recognize, or demonstrate by sketching, translation, reflection, and rotation. | Grade 7 |

| Data and Chance: Organization and Representation | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Read data from tables, pictographs, bar graphs, pie charts, and line graphs. | Tables, line graphs picture graphs – yes, ,Pie charts – no |
| 2. Organize and display data using tables, pictographs, bar graphs, pie charts, and line graphs. | Tables, line graphs Picture graphs – yes, Pie charts – no |
| 3. Compare and match different representations of the same data. | Not explicit |

# 8th Grade TIMSS Comparison

| Data and Chance: Data Interpretation | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Identify, calculate and compare characteristics of data sets, including mean, median, range, and shape of distribution (in general terms). | Grade 6. No – Mode. |
| 2. Use and interpret data sets to answer questions and solve problems (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points). | Yes |
| 3. Recognize and describe approaches to organizing and displaying data that could lead to misinterpretation (e.g., inappropriate grouping and misleading or distorted scales). | No |

| Data and Chance: Chance | |
|---|---|
| **TIMSS Topic** | **CCSS Comparison** |
| 1. Judge the chance of an outcome as certain, more likely, equally likely, less likely, or impossible. | No, not explicitly. |
| 2. Use data from experiments to predict the chances of future outcomes. | No, not as clear as it could be. |
| 3. Given a context, use the chances of a particular outcome to solve problems; determine the chances of possible outcomes (e.g., a particular face has a one-sixth chance of being on top after dropping a number cube). | Grade 7 |

**Who We Are**

A diverse team of over 20 individuals from across the country conducted this review. In addition to the team, many other individuals contributed feedback and input or supported the effort in other ways. Many of the team members have previously been involved in analyzing, reviewing, and developing math standards. The team consists of people who either now or at some point have been a parent, engineer, computer software engineer, policy analyst, math tutor, lawyer, and educator.

Among the educators, there are teachers who collectively have taught math at every grade level K-12. Additional educational roles represented on this team are elementary principals, curriculum consultants, a staff development coordinator, and school board members. One curriculum consultant was a full time employee of a state department of education and had responsibility for developing the state's K-8 science standards.

All participants in this review have a passion for seeing that the students in their respective states and across the country are provided the opportunity for a world class math education.