COMMENTS ON THE COMMON CORE STANDARDS FOR MATHEMATICS

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Submitted by the U. S. Coalition for World Class Math
http://usworldclassmath.webs.com/

I. Introduction

The Common Core State Standards lead off with Standards for Mathematical Practice. The introduction to the Standards reads:

> *The standards for mathematical practice rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*

To the casual observer, these words sound reassuring. For those who have been involved in the debate over how best to teach mathematics for the last two decades, this paragraph is extremely disturbing. NCTM's process standards have been interpreted and implemented so as to downplay the importance of procedures and algorithmic efficiency in the name of "understanding". It also favors finding more than one way to arrive at an answer that usually can be arrived at very simply in one way, and of eschewing word problems that provide the data that students will need to solve the problem in the belief that finding the data by themselves builds better problem solvers. We believe that the allegiance to the principles of the NCTM standards and ideology in Adding it Up will manifest itself in a student-centered, inquiry-based approach to math. We set out below attributes of the standards that are particularly weak and which lend themselves to such educational philosophy. As such, these standards in our opinion will diminish, not enhance, the mathematical proficiency and knowledge of students in K-12.

II. A Promise Made, A Promise Not Delivered

A. International Benchmarking

The CCSSO, NGA, and CCSSI started out with the following promise:

> *The standards will be research- and evidence-based, aligned with college and work expectations, include rigorous content and skills, and be internationally benchmarked.*

They are now saying that the standards "*Are informed by other top performing countries, so that all students are prepared to succeed in our global economy and society" and "Are evidence-based"*

Is that backpedalling on the internationally benchmark promise? There is a significant difference between what they promise and what they say they are delivering.

B. Fewer Standards

On page 3 of the standards document it says:

> *It is important to recognize that "fewer standards are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.*

Standards	**Grade Level**	**With Process Standards***	**Without Process Standards***
CCSS	K-8		315
California	K-7	424	329
California Green Dots	K-7		125
Washington	K-8	319	243
Indiana	K-8	438	251

*Process standards is used here to mean any of the standards many states have a separate strand for that may include standards for problem solving, reasoning, or mathematical processes. The CCSS addresses this in a section titled Standards for Mathematical Practice so they are not included in the standards count.

The CCSS may not necessarily be fewer, but they are acceptable in number. The number of standards does not tell the complete story. Standards from the states in the chart tend to be presented with great clarity, are fairly brief, and pedagogy free. This is also true of some of the CCSS. However, too many of the CCSS lack clarity, are too lengthy, and are loaded with pedagogy.

C. Other Promises

The CCSS document provides a Sample of Works Consulted at the end. The document does not present information showing how these standards are research and evidence based. How are these standards aligned with college and work expectations? The K-8 standards do seem to include rigorous content and skills even if not evenly addressed or appropriately placed.

These standards, in a number of places, are not presented in an appropriate sequential and hierarchical order. In the larger scheme, necessary content may be present, but presented at inappropriate grade levels.

III. Requiring Students to “Understand” and “Explain” a Concept

The use of the word “understand” as a leading verb in standards results in different interpretations by different people for different purposes. The use of the word “understand” has been reduced to nearly a third of its use in the March draft. Even with the reduction in use, the problems of its use still remain. The use of “understand” as leading verb robs a standard of necessary clarity and replaces it instead with a lack of specificity. While these standards may be assessed, the assessment may be different than the instruction provided as a result. Well-written standards should uniformly set clear and specific direction for instruction and assessment. The standards in the CCSS are written in such a way as to allow assessment development control and direction over the interpretation and meaning of the standards.

The word “understand” in some instances has been replaced with “explain” and raises concern. For example, under the Number System standards for sixth grade is the following:

> *1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9*

because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?"

It is indeed desirable to teach students the conceptual underpinnings of procedures, and we have no disagreement with that. We do not believe, however, that it is necessary to require students to then be able to recite the reasons why a particular procedure or algorithm works; i.e., to provide justification. At lower grade levels, some students will understand such explanations, but many will not. The key is whether they understand how such procedure is to be applied, and—as the above standard illustrates—what the particular procedure represents. The problems given as illustrations in the above standard provide evidence that students understand what fractional division means, without having to ask them to explain what the relationship is between multiplication and division.

As students progress through the grades, they acquire more procedural fluency, and ultimately more understanding. When they are in algebra classes and are able to use algebraic symbols, mathematical reasoning is increased because they have more tools with which to express mathematical ideas. At that point, it is perfectly reasonable to expect students to be able to show understanding by requiring them to solve specific problems. For example, students are able to demonstrate "understanding" of the derivation of the formula for the quadratic equation, by solving the equation $ax^2 + bx + c = 0$ using the method of completing the square.

The prevalent myth is that explaining an answer is inherently connected with understanding it. Thus, even with the decreased use of the word "understand", the belief that unexplained answers are "mere calculation" seems to pervade these standards. Students who do not apply math to real-life situations or demonstrate their strategies in words and pictures, however accurately they calculate the answers, are held to not understand underlying concepts. The ultimate result of such thinking may be that students who cannot calculate correct answers but can "explain" their thinking will get partial or even full credit for answers on tests. Most importantly, these standards in urging explanations, interpretations and understanding, via diagrams and explanations, fail to acknowledge that there are some extremely analytic children who for a variety of reasons cannot express themselves well in writing. (Some of these may have Asperger's syndrome or similar afflictions.) Many of these children can easily do math in their heads, and are able to solve very complex problems, but often will be unable to explain—in writing or verbally--how they arrived at their answers.

Asking for justifications, interpretations and explanations will manifest itself by asking students to provide two or three ways to solve a simple computation problem and asking them to explain their procedure in words or to draw pictures. (In fact, as the above example illustrates, students are expected to present visual fraction models to accompany a fraction division problem.) Enforced understanding is ultimately ineffective and most likely will result in students being required to memorize (by rote) an explanation they don't understand. And for a segment of the student population who understand math extremely well but because of their mental make-up cannot express mathematical ideas in writing or pictorially, they will be short-changed. They will likely get poor grades, be deemed unfit for honors or enrichment type classes in which they would likely benefit. Instead they are left behind and wrongly judged to be good "only" at computation, when in fact many are highly gifted and entitled to far more than an understanding-obsessed educational system is willing to provide. We believe these standards embody this very wrong way of thinking.

IV. Lack of Clarity, Inconsistent Skill and Concept Development, and Placement of Standards

The CCSS standards are poorly written. They are convoluted and inconsistent. These standards are not clear and for many will be difficult to use and understand. As for any promise to be fewer, there are places were they have four or five standards that could effectively be replaced by one clear and brief easily understood standard. There are many “fat standards” containing what could be broken into three or more distinct, but related, standards.

There are skills and concepts the CCSS develop in excruciating detail (do not equate that with clarity, because that detail can be a bear to figure out---and will be for primary teachers). There are skills and concepts the CCSS jump right into the middle of with no introduction or development. These standards expect students know and be able to apply previously undeveloped skills and concepts in the context of problem solving and word problems.

Some of the standards provide one or more examples. This often is led off with “For example,”. In the case of many of these standards, the example is essential to understanding the standard and in some cases could stand in place of the standard. This should not be the case. The standards should be self-explanatory and stand-alone.

Too many of the CCSS standards are laden with pedagogy.

A. Skill/Concept Development Example

Here are the CCSS standards from Grade 3 developing the concept of area.

Grade 3

> *Recognize area as an attribute of plane figures and understand concepts of area measurement.*

A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Relate area to the operations of multiplication and addition.

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems; represent whole-number products as rectangular areas in mathematical reasoning.

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c; use area models to represent the distributive property in mathematical reasoning.

Recognize area as additive; find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

The third grade standards develop in detail the concept of area but neglect to directly develop or derive the formula from that work. Here is the Grade 4 CCSS related standard. Why wait a year before using the formula for area?

Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

It is interesting to note that the Grade 4 standard above calls for students to apply the area and perimeter formulas for rectangles, no standards develop the concept, skill, or formula for perimeter. The standard presented below is the only grade 3 standard addressing perimeter.

CCSS Grade 3 Perimeter

Solve real-world and mathematical problems involving perimeters of polygons, such as finding the perimeter given the side lengths, finding an unknown side *length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.*

Why the discrepancy in concept and skill development? If such detailed concept and skill development is going to be in the standards, shouldn't it be there for all concepts and skills? As presented in these standards, there are a number of such discrepancies; area and perimeter were used as only one example. Such detailed concept and skill development in the standards are pedagogy loaded. The standards should be as pedagogy free as possibly. A standard presented as simple as "Calculate the perimeter and area of a rectangle using formulas" clearly states what students need to be able to do and leaves it up to the professional judgment of the teacher as to how to teach the concept and skill so students can successfully meet the standard.

Will such uneven topic development determine instructional time allocation and pacing?

B. Fat Standards and Pedagogy

Standards should be free of pedagogy as much as possible. Many of the standards in the CCSS contain pedagogy. The "fat standards" often contain pedagogy. Fat standards are ones that 1) could be broken down into multiple standards, 2) lead examples in with "For example" or "e.g.", or 3) both. While the pedagogy in many cases is used as an example, teachers and administrators will often interpret and apply it as a part of the standard. Consider this Grade 4 standard.

> *Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

This standard could effectively be broken into separate standards.

> *Compare two fractions with different numerators and different denominators.*
>
> *Recognize that comparisons are valid only when the two fractions refer to the same whole.*
>
> *Record the results of comparisons with symbols >, =, or <, and justify the conclusions.*

The "by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½" and "by using a visual fraction model" are pedagogy. The standards should be able to stand alone without embedded pedagogical examples. This standard could easily be interpreted as having a choice between creating common denominators or numerators and comparing to a benchmark fraction. With that being a choice, the CCSS never explicitly require students to find common denominators.

C. Questionable Placement of Standards

This standard is in the CCSS grade 7.

> *Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.*

Standards doing the same basic thing are found in other standard sets in grade 4, 5, and 6. Why the delay? Why not cover this as a logical extension of developing facility with decimals? This is something students can actually use in real life to figure a percent grade on their work, say if they got 17 correct out of 18. But that may not as important as it used to be if the students are graded with a 1, 2, 3, or 4 instead of letter grades. The terms rational numbers, repeating decimals and terminating decimals may not appear in the standards at lower grade levels in other states, repeating decimals and terminating decimals are terms used when students are taught to convert.

D. Standard Algorithms---Too Little, Too Late

The CCSS does contain standards addressing and requiring the use of standard algorithms for addition, subtraction, multiplication, and division. These standards are clear and straight forward. Unfortunately, the standards for addition, subtraction, division come too late. In addition to what is currently in place, there should be a standard in grade 2 requiring the use of the standard algorithm for addition and subtraction and one in grade 4 or 5 for division. The standard algorithm should be taught towards from the beginning. Various strategies may be helpful in understanding how the standard algorithms work but lack in efficiency. Students not oriented towards the standard algorithms for addition and subtraction in grade 2 may never transition from well practiced less efficient strategies in grade 4 when required to use the standard algorithm. This potentially sets students up for failure and remediation efforts wasting valuable instructional time. The same can be said for division at the appropriate grade levels.

E. A Fractional Preparation

The March draft left out many important foundational skills critical for success in authentic algebra and beyond. Some, not all, of those skills have been added to the final CCSS. It is questionable if what has been added will adequately prepare students to successfully employ fraction skills in algebra or higher level math. Terms that were missing but are now present include common denominator, least common multiple, greatest common factor, factor pairs, and prime. Fraction related terms still missing are least common denominator, prime factors, simplifying, reducing, or reduce to lowest terms. The inclusion of a term does not mean the skill is adequately developed, if developed at all, or placed appropriately.

Common denominator is mentioned once in the standards. This occurs in the Grade 4 standard examined in the *Pedagogy and Fat Standards* section. Common denominator is not mentioned again nor are students ever explicitly required to find a common denominator.

In Grade 5, this standard has students adding and subtracting fractions with unlike denominators without the benefit of knowing about or using least common multiple (or least common denominator).

> *Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

In Grade 6 students find greatest common factors and least common multiples.

> *Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).*

This is work done in isolation with no connection made with what the greatest common factor and least common multiple might be used for. There is some important skill development here with no value or utility shown. Is it possible these skills are inappropriately placed? Is it possible, if presented and connected earlier, these skills might help students be more successful and efficient in adding and subtracting fractions with unlike denominators?

In the *Lack of Clarity, Inconsistent Skill and Concept Development, and Placement of Standards* section, the standards used as examples from K-8 are representative of other standards exhibiting similar characteristics.

V. CCSS High School Mathematics Standards

On page 84, at the very end of the Common Core Mathematics Standards document, is this statement:

> "The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well."

This statement makes clear what a review of the actual standards indicates – these "standards" appear to be a summary of what a graduating high school senior should know, and not a course-by-course statement of what content a student should know after taking that course. As such, the standards are useless for constructing assessments for use as students proceed through high school mathematics.

It looks like the standards are incomplete and that they were issued to meet a deadline with the understanding that a complete set might be issued later. But it has not been made clear whether the "pathways" described on page 84 will actually be standards for individual courses. States that are signing on to adopt these standards are buying a pig in a poke.

The existing standards are strong in most areas but weak in trigonometry, omitting an entire area that is usually called "simplifying trigonometric expressions" or "proving identities." According to these standards, a student need prove only the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ and the addition and subtraction formulas for sine, cosine, and tangent.

In one area the Common Core standards are superior to the Washington standards: statistics and probability are in a separate category; in the Washington standards, they are included in algebra, where they do not belong.

The organization of the Common Core standards is uneven and illogical. Some examples:

> 1. There are two cluster headings under "The Real Number System" domain. One is "Extend the properties of exponents to rational exponents;" the other is "Use properties of rational and irrational numbers." In comparison, the Washington State Standards, under the title "Numbers, expressions, and operations," have the standard "Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line." This Washington standard, which goes to the core of what real numbers are, is missing from the Common Core standards. Can anyone believe that the two

Common Core cluster headings summarize what students should know about the real numbers? The cluster heading about rational exponents doesn't really belong under the "Real Number System" domain anyway. While it is true that nonnegative real numbers with rational exponents are themselves real numbers, negative real numbers with certain rational exponents are not real numbers. It might make sense to include this cluster heading under the "Complex Number System" domain.

2. The Common Core standards have an entire separate section with no standards at all! This section, called "Modeling," merely refers to specially-marked standards in other sections.

3. Each section of the standards contains the exact same list of "mathematical practices." These could be called "guidelines for use when doing mathematics." The list is reasonable, but it is not a list of "standards." Standards should be measurable, and these items are not.

To summarize: the Common Core high school mathematics standards are poorly organized; sometimes they are not "standards" in that they are not measurable; and they cannot be used to develop assessments for mathematics courses. Perhaps the "pathways," when released, will be of more use.

VI. Conclusion

In a side-by-side comparison of the CCSS with standards from California, Indiana, and Washington, it is apparent the standards from CA, IN, and WA are clear, concise, and written in a consistent manner. For the most part, an elementary teacher would be able to glance and go teach with the CA, IN, or WA standards. That is not the case with the CCSS standards. As result, along with concerns addressed in this review, the CCSS rate as a good C+ to B- set of standards.

These comments about the Common Core State Standards in this review focused on areas of concern. States adopting these standards need to be aware of these concerns. The standards are impressive in their presentation and as such it is difficult for many to see any areas of concern.

In brief, here are some major areas of concern:

- The lack of clarity and specificity will lead to a lack of uniformity in instruction and assessment. The requirement to “understand” and “explain” contribute to this lack of clarity and specificity.
- The inappropriate placement of standards, including the delayed requirement for standard algorithms, may hinder student success and waste valuable instructional time.

- The uneven treatment of topics establishes a hierarchy of importance that may be unintended. This may result in an inefficient use of instructional and practice time with too much time devoted to some topics and not enough time devoted to other topics.
- The high school standards are not well organized and some topics are not addressed to the extent they should be.

After thoroughly examining these standards some big questions come to mind. Are these standards really “substantially more focused and coherent”? Will these standards make the U.S. internationally competitive? For those states adopting these standards, time will provide answers. We do not have time to waste, however. And, as we indicated in the introduction, two decades of NCTM's standards have led to a serious downfall in how math has been taught in the U.S. For us, the answer is clear. These standards are not internationally competitive. At best, some students will be led to "appreciate" math; at worst, many students will be vastly short-changed. This is an opportunity cost this country can ill afford.

About the Reviewers

Although many individuals provided input for this review, there were three main contributors. The three main contributors have been involved in analyzing, reviewing, and developing math standards. Among the three, two have written articles about math education. Among the reviewers, one is a policy analyst and two are educators. The policy analyst worked for a U.S. Senator and did research in the area of math education. The experience of the educators includes over 35 years in public education. This experience includes teaching math at every grade level K-12 with one focusing on elementary and middle grades and the other focusing on high school and teaching calculus. Additional educational experience of the reviewers includes experience as an elementary principal, curriculum consultant, and a staff development coordinator.